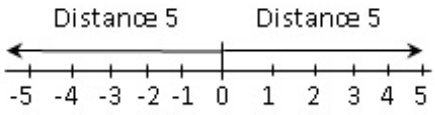
I can say with reasonable certainty that until and unless you are a quant jock, you hate Inequalities (especially when they present themselves in DS questions). I can also say the same thing about Modulus (perhaps, not to the same extent!). So I can imagine what you feel when you come across a DS question with both Modulus and Inequalities! I am also certain that a small part of you, a very small part indeed, is secretly thrilled to see such a question because it implies that you are doing well in the exam and the software is getting jittery and trying to give you harder and harder questions. But wouldn't it be something if you could crack the question in a minute and then say, "What else you got?"

It is not very difficult to do that, in my opinion. GMAT is not about solving complicated equations/inequalities. It is about using your bare fundamentals, quietly, inconspicuously… I am reminded of Harry Potter and the Half-Blood Prince here (Yes, I am a Harry Potter fan and this time, I cannot blame my little daughter for it!). Dumbledore and Harry go to some cave to get a Horcrux and Dumbledore quietly works his way through, feeling around for magic. At that point, the author says, "He had never seen a wizard work things out like this, simply by looking and touching; but Harry had long since learned that bangs and smoke were more often the marks of ineptitude than expertise." I think it is quite an appropriate analogy. GMAT is not about booms and bangs and fancy equations; it is more ethereal; you have to be more in touch with your concepts…

Anyway, I don't want you to run away if you came here looking for some solid fundamentals. Let me start by saying that inequality just implies less than or greater than. Don't try to read too much into it. Modulus stands for the distance of a point from 0 on the number line. When I say |x| = 5, it means x is a point at a distance 5 from 0 on the number line. Take a minute to go through it again: x is a point at a distance 5 from 0 on the number line. Which points are at a distance 5 from 0 on the number line?

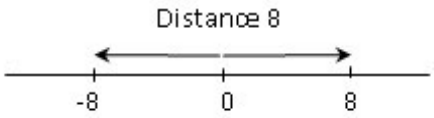
$|x| = 5$



So x = 5 or -5

Points 5 and -5. So x can take two values: 5 or -5. Similarly, if |x| = 8, x can take two values x = 8 or x = -8.

$|x| = 8$



So x = 8 or -8

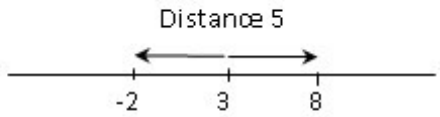
$|x| = -2$

Not possible because distance cannot be negative

Now, what if you have something like |x-3| = 5? What does this mean? The distance you are looking for is still 5. The only difference is that the distance is from the point 3 now.

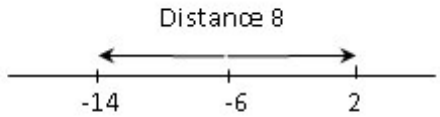
Let's look at the figure to see what values x can take.

$|x-3| = 5$



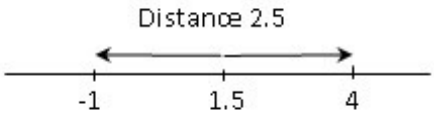
So $x = 8$ or -2

$|x+6| = 8$



So $x = 2$ or -14

$|2x-3| = 5 \rightarrow 2*|x-1.5| = 5 \rightarrow |x-1.5| = 2.5$

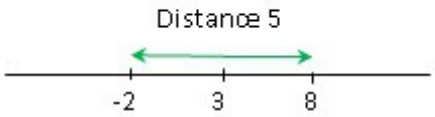


So $x = -1$ or $x = 4$

In the last example above, we say that twice the distance from 1.5 is 5. So we want the point that is 2.5 away from 1.5.

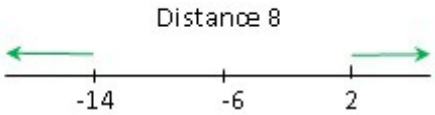
If this makes sense to you, we could include an inequality sign here. Things will still be no different. For example, what values can x take if $|x-3| < 5$. We need those points where distance from point 3 is *less than* 5. At all the points depicted by the green line in the diagram below, the distance from 3 is less than 5. Point 4 is at a distance 1 away from 3, point 7 is at a distance 4 away from 3, 0 is at a distance 3 away from 3 and so on… Point 8 is at a distance 5 away from 3 so all points lying between 3 and 8 are at a distance less than 5 from 3. Point 3 is at a distance 0 from 3 so it is also included but 8 is not since the inequality doesn't have an equal to sign.

$|x-3| < 5$



So $-2 < x < 8$

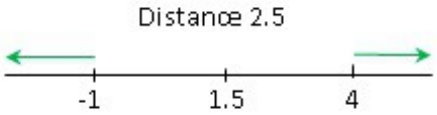
$|x+6| \geq 8$



Distance from -6 should be greater than 8.

So $x \geq 2$ or $x \leq -14$

$|2x-3| > 5 \rightarrow |x-1.5| > 2.5$



So $x < -1$ or $x > 4$

This is an extremely efficient way of working with inequalities + modulus.